Table 7 Kinetic parameters of HC-434 for different sample masses

| Sample mass (mg) | Coats-Redfern E | Coats-Redfern A | Coats-Redfern r | MacCallum-Tanner E | MacCallum-Tanner A | MacCallum-Tanner r | Horowitz-Metzger E | Horowitz-Metzger A | Horowitz-Metzger r |
|---|---|---|---|---|---|---|---|---|---|
| 1.25 | 111.7 | $2.18\times10^5$ | 0.9952 | 115.7 | $4.40\times10^5$ | 0.9960 | 134.2 | $9.17\times10^6$ | 0.9980 |
| 2.5 | 108.8 | $1.22\times10^5$ | 0.9932 | 112.9 | $2.48\times10^5$ | 0.9943 | 131.8 | $5.54\times10^6$ | 0.9972 |
| 5.1 | 107.2 | $9.35\times10^4$ | 0.9918 | 111.2 | $1.88\times10^5$ | 0.9982 | 131.7 | $5.36\times10^6$ | 0.9963 |
| 7.5 | 110.8 | $1.57\times10^5$ | 0.9900 | 115.0 | $3.22\times10^5$ | 0.9916 | 135.3 | $8.91\times10^6$ | 0.9939 |
| 10.2 | 95.8 | $1.36\times10^4$ | 0.9985 | 99.6 | $2.06\times10^4$ | 0.9946 | 120.8 | $8.65\times10^5$ | 0.9976 |
| 14.9 | 102.8 | $3.70\times10^4$ | 0.9901 | 106.9 | $7.50\times10^4$ | 0.9918 | 128.1 | $2.33\times10^6$ | 0.9957 |
| 20.5 | 105.5 | $6.03\times10^4$ | 0.9891 | 109.6 | $1.22\times10^5$ | 0.9909 | 131.8 | $4.50\times10^6$ | 0.9900 |

The units of $E$, $A$, and the heating rate given in the tables are kJ mole$^{-1}$, s$^{-1}$, and °C min$^{-1}$, respectively. From these tables, the following observations can be made:

1) The correlation coefficients are close to unity indicating near-perfect fits.

2) The kinetic parameters calculated with the Horowitz-Metzger method are higher than those with the other two methods; this is due to the approximation technique used in the integration of the former. Similar observation has been made earlier.[4]

3) The kinetic parameters are not significantly affected by either heating rate or sample mass, the deviations being about 10%, which is within the scatter usually observed in TG experiments.[6]

4) Irrespective of the nature of the terminal groups and the method of preparation, the energy of activation for the thermal decomposition of all the five resins is around 110 kJ mole$^{-1}$ and the preexponential factor is of the order of $10^5$ s$^{-1}$, showing that the decomposition kinetics are dependent only on the polymer backbone (which is the same in all the cases studied). The values of $E$ and $A$ are in close agreement with the reported values for CTPB resin.[7]

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# Measurement of Nutation Parameters for the Geostationary Telecommunication Satellite Sirio

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## Introduction

SIRIO 1 was launched by a Thor-Delta rocket from Cape Kennedy on August 25, 1977. Mission control was at Goddard Space Flight Center (NASA). Sirio 1 is a geostationary, spin-stabilized satellite with a despun super-high-frequency (SHF) antenna, stationed on the equator, 15° W of Greenwich. The spacecraft is cylindrical in shape, with the Apogee Boost Motor (ABM) nozzle protruding at one end and the SHF antenna at the other. The principal axis of inertia coincides with the cylinder geometric axis $z$. The spacecraft is dynamically stable around the $z$ axis. An accelerometer and a nutation damper (NUD) are placed inside the spacecraft. In this paper the analysis of accelerometer telemetry data is described for the purpose of measuring nutation parameters. The accelerometer is mounted parallel to the spacecraft geometric axis $z$ and measures the acceleration component parallel to that direction in the spacecraft reference system. The algorithms used to analyze rectified acceleration data and acceleration sign are described. The two cases of negligible and non-negligible bias are discussed.

The simple devices that prevent noise and data-flow gaps from affecting measurements are described. A method is found to analyze the sign which can recover a period as short as 1.6 s with a sampling time of 4 s, with an ambiguity on the measured value which is resolved by knowledge of a theoretically expected value or by comparison with the period calculated from amplitude. The program was tested with simulated data before launch, and worked as expected during flight. Measurements are obtained for nutation period, ratio of moments of inertia, nutation amplitude, damping time, misalignment, and bias. The spacecraft behaves as expected and nutation is efficiently damped by NUD and fuel sloshing.

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### Nutation

In order to achieve station, various maneuvers have to be performed which affect not only spacecraft orbit and/or attitude, but also its spinning motion by causing nutation. In case of small nutation amplitude, the acceleration parallel to the geometric axis $z$ of a cylindrically symmetric, almost rigid, spinning body can be described as a damped oscillation:

$$a=\lambda(2-\lambda)\omega_0^2R\vartheta_0e^{-t/\tau}\cos[(\lambda-1)\omega_0t]+\mu\omega_0^2R+b$$

This equation is easily derived[1] from basic motion laws. The spin rate $\omega_0$ (nominally $\omega_0=2\pi\cdot 90$ rpm, period $p_0=0.666$ s) is measured by on-line programs that process telemetry data. The ratio of moments of inertia is $\lambda=I_z/I_x=1+p_0/p$ ($I_x=I_y$) and is expected to be $\lambda\sim 1.1$ at launch, $\lambda\sim 1.3$ after ABM firing (ABMF). Correspondingly, the expected nutation frequency $\omega=(\lambda-1)\omega_0$ is $\omega\sim 2\pi\cdot 9$ rpm ($p\sim 6.66$ s) at launch, $\omega\sim 2\pi\cdot 27$ rpm ($p\sim 2.22$ s) after ABMF. The accelerometer is mounted at a distance $R=0.624$ m from the center, with an expected misalignment $\alpha=0.1$ deg, where $\mu=\sin\alpha$. The initial aperture $\vartheta_0$ of the nutation cone is damped with a time constant $\tau$ by nonrigidity (NUD and fuel sloshing). The expected value is $\tau\sim 180$ s. An electrical bias $b$ can be present. Given these expected values, a nutation amplitude $\vartheta_0\sim$ 1 deg corresponds to an acceleration $a\sim 0.1g$.

### Accelerometer Telemetry Data

Telemetry data are broadcast by the satellite via VHF and received by Earth stations, then relayed to Mission Control, where they are recorded on the telemetry tape and handed over to on-line programs for real-time computations (e.g., attitude, spin rate, etc.). In this work, data are read from the telemetry tape and the program is run off-line. The acceleration data channel has a full scale of $0.291g$ (which corresponds to about 3 deg nutation amplitude) and 8 bits (count from 0 to 255). The full scale corresponds to 250 counts. Precision is $\pm 5$ counts. Each of the two methods described in the following sections uses one of the two kinds of available data: absolute value of acceleration, sampled every 0.5 s, and acceleration sign, sampled every 4 s. These data are subject to digital errors and gaps in the transmission, which have to be filtered. Expected nutation periods range from 2 to 7 s, and for the rectified amplitude (negligible-bias case, Fig. 1a) from 1 to 3.5 s. A period of 1 s is the shortest period which can be measured by sampling every 0.5 s. A special method instead is devised to obtain the period from the sign, as this is sampled less than twice every nutation period, or even less than once. If the initial oscillation amplitude is larger than the bias, this is negligible (Fig. 1a), the sign changes and the sampled values are treated as if generated by a rectified, damped oscillation. Fast damping makes the sign available for about only 1 min after the maneuver. The amplitude rapidly becomes smaller than the bias: the latter is non-negligible (Fig. 1c), the sign does not change and the oscillation around the bias is clearly visible for a few minutes after the maneuver. Most measurements are done in this situation by using the amplitude method: this gives the more precise results, owing to the longer observation time. Between the above two cases there is an intermediate one (Fig. 1b), where the rectified sum of the negative half-wave plus the bias does not reach up to the average of the curve. In this case the sign is still available, while the bias can be considered negligible for calculating amplitude and damping time, but not for the period.

#### Calculation of Nutation Parameters from Rectified Acceleration Data

Autocorrelation gives results that depend on $T$, where $T$ is the chosen time interval length. Fourier analysis gives a distribution of amplitude coefficients $c_n$ corresponding to periods $p_n=T/n$ ($n$ is an integer). These results are not readily useful and these methods are not used. Superimposing expected and measured values and trying to determine parameters by least squares leads to numerical instability. The period is calculated as follows. Given a set of data, the average is computed and a histogram is made of periods (with respect to the average) with a step of 0.5 s. A distribution of half-periods in case of negligible bias and whole periods in case of non-negligible bias is thus obtained. The period is computed by a weighted average on the peak and the two adjacent values in the distribution. This rules out abnormally long or short "periods" due to digital errors or data-flow gaps. Calculations performed with simulated data, with digital errors as frequent as 1 every 10 data points, yield results accurate to two significant figures. With more realistic error rates, 1 every 100 or more data points, precision increases to three significant figures. Two different algorithms for calculating amplitude and damping time are used, depending on the bias. In case of negligible bias (Fig. 1a) the average $\bar{A}$ is directly related to the amplitude $A$ by

$$\bar{A}=\frac{1}{\pi}\int_0^\pi A\sin t\,\mathrm{d}t=\frac{2}{\pi}A$$

Averaging greatly reduces the weight of errors. The total time interval is divided into three intervals and for each the average is calculated. Owing to the damping factor, the envelope amplitude $A=(\pi/2)\bar{A}$ is attributed to the instant central of the time interval considered. Initial amplitude and damping time are computed by least squares. In case of non-negligible bias (Fig. 1c) the peak-to-peak amplitude is given by the half-width of the histogram of data. This prevents digital errors from affecting the result. A histogram is calculated for each minute of data (120 points), the resulting amplitude is attributed to the instant central to the minute considered, and initial amplitude and damping time are computed by least squares.

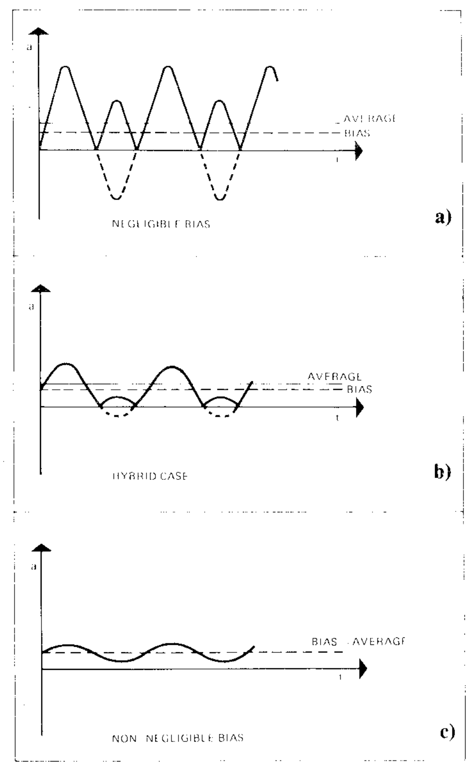


**Fig. 1 Accelerometer bias.**

#### Calculation of Nutation Period from Acceleration Sign

The only information that can be possibly obtained from this channel is the nutation period $p$. No attempt is made to correlate sign and rectified acceleration signal. The two are treated as independent sources of information. The sign, when available (Figs. 1a and 1b), gives an independent measurement of the nutation period, which is consistent with the one measured from amplitude. Superposition of a square wave or a sine wave on simulated sequences of pluses and minuses fails to yield reliable results. The method leading to a satisfactory algorithm is described in the following. Assume that $p$ is close to the sampling time $s$:

$$p = s + \Delta t \tag{1}$$

If $p = s$, there results an uninterrupted sequence of, say, all pluses. But if $p \neq s$, the difference $\Delta t$ will cause the sign to change after $n$ samples, if $n$ is such that

$$n|\Delta t| = p/2 \tag{2}$$

In the telemetry channel, therefore, there are sequences of pluses and minuses with a length $n$, which can be measured. If $s$ is given and $n$ is measured, $|\Delta t|$ is derived from Eqs. (1) and (2) by eliminating $p$:

$$2n|\Delta t| = s + \Delta t \tag{3}$$

$$|\Delta t| = s/(2n-1) \quad \text{if} \quad \Delta t > 0 \tag{4}$$

$$|\Delta t| = s/(2n+1) \quad \text{if} \quad \Delta t < 0 \tag{5}$$

There is no way of knowing if $\Delta t$ is positive or negative. Therefore, two values for $p$ are obtained from Eqs. (1), (4), and (5):

$$p = 2ns/(2n-1) \quad \text{if} \quad \Delta t > 0 \tag{6}$$

$$p = 2ns/(2n+1) \quad \text{if} \quad \Delta t < 0 \tag{7}$$

If $|\Delta t|$ is very small, $n$ may be larger than the number $N$ of samples considered. In this case it can be estimated:

$$|\Delta t| \leq s/2N \tag{8}$$

Minimum $n = 1$ implies max $\Delta t = s$, i.e., max $p = 2s$, and min $\Delta t = -s/3$, i.e., min $p = 2s/3$. The range of validity for $s = 4$ s is max $p = 8$ s, min $p = 2.66$ s, which is in the range of interest. In order to cover a possible $p \sim 2$ s, assume $p \sim s/2$:

$$2p = s + \Delta t \tag{9}$$

The sign changes after $n$ samplings:

$$n|\Delta t| = p/2 \tag{10}$$

Eliminating $p$ obtains

$$4n|\Delta t| = s + \Delta t \tag{11}$$

**Table 1 Results**

| | Before ABMF | | After ABMF | |
|---|---|---|---|---|
| Nutation period $p$, s | 5.20 | 5.17 | 2.26 | 2.22 |
| Spin rate $\omega_0/2\pi$, rpm | 84.07 | 84.20 | 84.53 | 87.68 |
| Spin period $p_0$, s | 0.714 | 0.713 | 0.710 | 0.684 |
| Ratio of moments of inertia $\lambda = I_z/I_x = 1 + p_0/p$ | $1.1375 \pm 0.0005$ | | $1.311 \pm 0.006$ | |
| time after maneuver | $t=0$ | $t=4$ min | $t=8$ min | |
| Nutation amplitude $\vartheta$, deg | 0.2 | 0.02 | <0.01 | |
| Acceleration $a$, $g$ | 0.02 | 0.002 | <0.001 | |
| Nutation damping time $\tau$, s | 50 | 150 | | |

Therefore

$$|\Delta t| = \frac{s}{4n-1}; \quad p = \frac{2ns}{4n-1} \quad \text{if} \quad \Delta t > 0 \tag{12}$$

$$|\Delta t| = \frac{s}{4n+1}; \quad p = \frac{2ns}{4n+1} \quad \text{if} \quad \Delta t < 0 \tag{13}$$

The limits of validity are min $p = (2/5)s$, max $p = (2/3)s$. For $s = 4$ s, min $p = 1.6$ s, max $p = 2.66$ s. There are four intervals for each of which a measure of $n$ gives the corresponding value of $p$:

```
          (1)  (2)  (3)        (4)
+-------+----+----+----+---------+-->   p (units s)
0       2/5  1/2  2/3   1         2
```

For $s = 4$ s,

```
          (1)  (2)  (3)        (4)
+---------+----+----+--+---------+-->   p (s)
0        1.6   2   2.66 4         8
```

Only knowledge of a theoretically expected value, or comparison with the period calculated from amplitude, can resolve the ambiguity. The value of $2n$ is measured from complete sequences of pluses and minuses in telemetry (including one change of sign) to compensate for the asymmetry introduced by the bias (Figs. 1a and 1b). Noise or errors superimposed on telemetry or gaps in the data flow can introduce wrong signs, interrupt a positive or negative sequence and cause a smaller $n$ to be measured. These effects are avoided by using the following method. A histogram of values of $n$ is made. A weighted average on the peak and the two adjacent values in the distribution eliminates spurious $n$'s and gives the value of $n$ that is introduced in Eqs. (6), (7), (12), and (13). Numerical calculations with simulated data recover the period with an accuracy of the order of 1%.

### Results

The results are presented in Table 1. Damping time $\tau$ is not constant, but is smaller for larger amplitudes. This means that the damping mechanism (NUD + fuel sloshing) is nonlinear, since $\tau$ is calculated by assuming exponential decay ($e^{-t/\tau}$). Given spacecraft and NUD characteristics, $\tau \sim 180$ s is expected. The lower values measured indicate that an important role is played by fuel sloshing. Residual nutation is lower than the resolution (one count), which is in turn smaller than the tolerance limit of $0.005g$ or 0.05 deg. The accelerometer misalignment $\mu\omega_0^2 R$ + electrical bias $b$ for ABMF is $0.006g$, for +2 days is $0.008g$, and for +5 days is $0.009\ g$. The total bias increases with spin rate. The tolerance limit of $0.01g$ (0.1 deg) is never exceeded.

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